KINGSWAY CHRISTIAN COLLEGE

MATHS DEPARTMENT

Course: Mathematics Methods Year 12

Assessment Task: Test 4 – Logarithms

Student Name: Sol key.

Date: 26th June 2017

Assessment Score: ________ / 40

Year Score: ________

Comments: ________

Teacher signature: ________

Parent/ Guardian signature: ________

Comments: ________

METHODS YEAR 12 **Test 4 2017**

Logarithms

Resource Free **Time: 35 mins** **Marks: / 40**

No notes or calculators allowed for this section.

Question 1 **(5 marks)**

Evaluate the following, giving your answer as a single log term:

$$\frac{(\log 5 - \log 3)^2}{\log \frac{3}{5}}$$

$$= \frac{(\log 5 - \log 3)(\log 5 - \log 3)}{-\log\left(\frac{5}{3}\right)} ✓$$

$$= \frac{(\log 5 - \log 3)\cancel{(\log 5 - \log 3)}}{-\cancel{(\log 5 - \log 3)}} ✓$$

$$= \log 3 - \log 5 ✓$$

$$= \log \frac{3}{5} ✓ \quad \text{OR} \quad -\log \frac{5}{3}$$

Question 2 **(9 marks)**

Solve each of the following equations. Leave answers in logarithmic form where necessary.

(a) $2^{x-3} = 5^{2x+1}$ (4 marks)

$$\therefore \log 2^{x-3} = \log 5^{2x+1}$$

$$\Rightarrow (x-3)\log 2 = (2x+1)\log 5 ✓$$

$$x\log 2 - 3\log 2 = 2x\log 5 + \log 5 ✓$$

$$x\log 2 - 2x\log 5 = \log 5 + 3\log 2$$

$$x(\log 2 - 2\log 5) = \log 5 + 3\log 2 ✓$$

$$x = \frac{\log 5 + 3\log 2}{\log 2 - 2\log 5} ✓$$

OR $\frac{1 + \log 4}{\log\left(\frac{2}{25}\right)}$

OR $\frac{\log 40}{\log\left(\frac{2}{25}\right)}$

(b) $3^{2x+1} - 5(3^x) - 2 = 0$ (5 marks)

$3.3^{2x} - 5.3^x - 2 = 0$

let $3^x = c$

$\therefore 3c^2 - 5c - 2 = 0$

$(3c+1)(c-2) = 0.$

$3c = -1$ or $c = 2$

$c = -\frac{1}{3}$ or $c = 2$.

$\therefore 3^x \neq -\frac{1}{3}$ or $3^x = 2$

$\downarrow$

N.A.

$\underrightarrow{x = \log_3 2}$.

-1 if you didn't reject the $-\frac{1}{3}$.

Question 3 **(5 marks)**

If $\log_{10} 2 = x$ and $\log_{10} 3 = y$. Express the following in terms of x and y

(a) $\log_{10} 0.6$ (2 marks)

$\log_{10} \frac{6}{10}$

$= \log_{10} \frac{3\times 2}{10}$

$= \log_{10} 3 + \log_{10} 2 - \log_{10} 10$

$= y + x - 1.$

(b) $\log_{10} 45$ (3 marks)

$= \log_{10} (3^2 \times 5)$

$= \log_{10} 3^2 + \log_{10} 5$

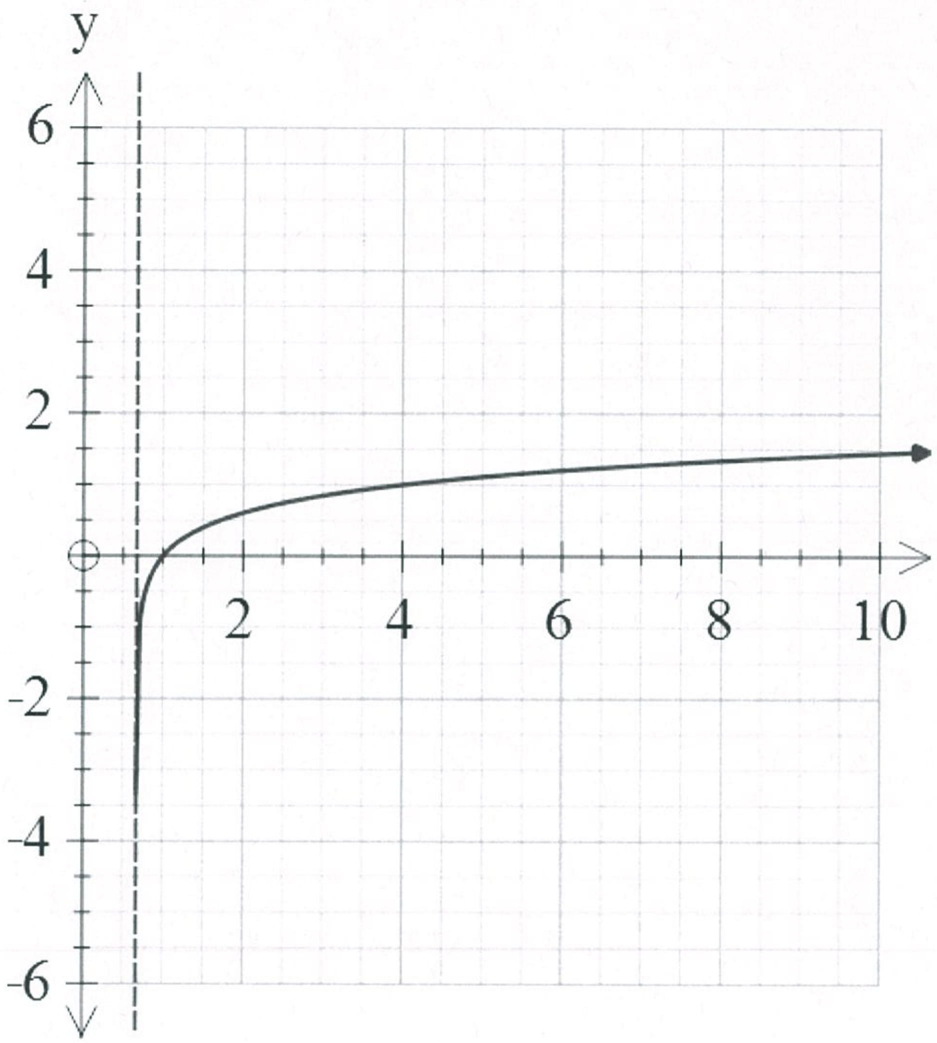
$= 2\log_{10} 3 + \log_{10} \frac{10}{2}$

$= 2\log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= 2y + 1 - x.$

Question 4 **(3 marks)**

The function $f(x) = \log(bx - 2)$ is drawn below.



(a) Determine the value of b. (2 marks)

$y = \log(bx - 2)$

through $(4,1)$ $1 = \log(4b - 2)$

$10^1 = 4b - 2$

$\therefore 4b = 12$

$\underline{b = 3}$

(b) Use the graph to approximate the solution to $\log(bx - 2) = 1$ (1 marks)

$\log(3x - 2) = 1$

$10^1 = 3x - 2$

$12 = 3x$

$\underline{x = 4}$

Question 5 **(3 marks)**

If $x = \frac{1}{\sqrt{3}}$, show that $\log(1 - x^4) - \log(1 - x) - \log(1 + x) = 2\log 2 - \log 3$.

$$LHS = \log \frac{(1-x^4)}{(1-x)(1+x)}$$

$$= \log \frac{(1+x^2)(\cancel{1+x})(\cancel{1-x})}{(\cancel{1-x})(\cancel{1+x})}$$

$$= \log(1+x^2)$$

$$= \log\left(1 + \left(\tfrac{1}{\sqrt{3}}\right)^2\right)$$

$$= \log\left(1 + \tfrac{1}{3}\right)$$

$$= \log\left(\tfrac{4}{3}\right)$$

$$= \log 4 - \log 3$$

$$= 2\log 2 - \log 3$$

$$= RHS.$$

Question 6 **(4 marks)**

State the following as y in terms of x

$$2\log_2(xy) = 5\log_2 x$$

$$\log_2 (xy)^2 = \log_2 x^5$$

$$\therefore (xy)^2 = x^5$$

$$x^2 y^2 = x^5$$

$$y^2 = \frac{x^5}{x^3}$$

$$y^2 = x^3$$

$$y = \sqrt[2]{x^3} \quad \text{or} \quad x^{3/2}$$

if $\pm$ then -1 mark → as $x > 0$ then y also > 0

Question 7 **(9 marks)**

Differentiate each of the following with respect to x.

(a) $y = \sqrt{x}\ \ln\left(\frac{x}{3}\right)$ (3 marks)

$$y = x^{\frac{1}{2}} \times \ln\left(\frac{x}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \cdot \ln\left(\frac{x}{3}\right) + x^{\frac{1}{2}} \times \frac{1}{\frac{x}{3}}$$

$$= \frac{\ln \frac{x}{3}}{2\sqrt{x}} + \frac{3}{\sqrt{x}}.$$

(b) $y = \ln\left[\frac{(x+4)^2}{(3x-1)}\right]$ (3 marks)

$$y = 2\ln(x+4) - \ln(3x-1)$$

$$\frac{dy}{dx} = \frac{2}{x+4} - \frac{3}{3x-1}. \quad \text{OR} \quad \frac{3x-14}{(x+4)(3x-1)}.$$

(c) $y = \frac{\cos^2 x}{\ln x}$ *(do not simplify)* → Quotient Rule. (3 marks)

$$\frac{dy}{dx} = \frac{-2\sin x\cos x(\ln x) - \frac{1}{x}\cos^2 x}{(\ln x)^2}$$

Question 8 **(3 marks)**

The tangent to the curve $y = \ln(kx - 1)$ has a gradient of 1 when $x = 2$. Determine the value of k.

$$\frac{dy}{dx} = \frac{k}{kx-1}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{k}{2x-1} = 1$$

$$k = 2k-1$$

$$1 = k$$

Question 9 **(2 marks)**

Determine the following anti-derivative, simplifying your answer using logarithmic laws if necessary:

$$\int \frac{5e^{-2x}}{1+e^{-2x}}\,dx$$

$$= 5\int \frac{e^{-2x}}{1+e^{-2x}}\,dx$$

$$= -\frac{5}{2}\int \frac{-2e^{-2x}}{1+e^{-2x}}\,dx$$

$$= -\frac{5}{2}\left[\ln\left|1+e^{-2x}\right|\right] + C$$